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2002 Commun. Theor. Phys. 37 583

(http://iopscience.iop.org/0253-6102/37/5/583)

Commun. Theor. Phys. (Beijing, China) **37** (2002) pp 583–588
Vol. 37, No. 5, May 15, 2002

Energy Level Statistics in Particle-Rotor Model*

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(Received September 14, 2001; Revised October 19, 2001)

Abstract *Energy level statistics of a system consisting of six particles interacting by delta force in a two-j model coupled with a deformed core is studied in particle-rotor model. For single-j shell ($i_{13/2}$) and two-j shell ($g_{7/2}+d_{5/2}$) the exact energies for our statistical analysis are obtained from a full diagonalization of the Hamiltonian, while in two-j case ($i_{13/2}+g_{9/2}$) the configuration truncation is used. The nearest-neighbor distribution of energy levels and spectral rigidity are studied as the function of spin. The results of single-j shell are compared with those in two-j case. It is showed that the system becomes more regular when single-j space ($i_{13/2}$) is replaced by two-j shell ($g_{7/2}+d_{5/2}$) although the basis size of the configuration space is unchanged. The degree of chaoticity of the system, however, changes slightly when configuration space is enlarged by extending single-j shell ($i_{13/2}$) to two-j shell ($i_{13/2}+g_{9/2}$).*

PACS numbers: 21.10.-k, 05.45.+b, 24.60.Lz, 21.10.Re
Key words: particle-rotor model, spectral statistics, the nearest-neighbor distribution, spectral rigidity

1 Introduction

Quantum chaos in many-body systems was studied mostly from the viewpoint of level statistics which displays a clear relation to the notion of classic chaos.[1] For quantal systems, corresponding to classical regular systems, the spectral statistics ($P(s)$ and $\triangle_3(L)$) follow the Poisson ensemble while for systems corresponding to chaotic ones the Wigner ensemble is followed.

Fluctuating properties of quantal systems fall into one of the four universality classes of Dyson and Mahta depending on the space-time symmetries of the Hamiltonian. In general, various approximations to the Hamiltonian are invoked in different nuclear models. The aim of these approximations is usually to obtain simplified calculations by throwing away certain symmetries. This may in turn have significant effects on the chaotic behavior of the system.

Particle-rotor model (PRM) and cranking shell model (CSM) are the appropriate realistic nuclear models for studying the coupling of the single-particle degrees of freedom to the collective motion through Coriolis force. Recently there is much discussion[2–5] about the difference between PRM and CSM. In Ref. [5], PRM and CSM were compared from the viewpoint of spectral statistics. This comparison, however, was made in the single-j shell space. This paper is to study in PRM if there are any changes about the conclusions of Ref. [5] when the configuration space is changed. The first step is to replace the single-j space ($i_{13/2}$) by a two-j space ($g_{7/2}+d_{5/2}$), and the basis size is unchanged. The second step is to enlarge the configuration space by extending single-j shell ($i_{13/2}$) to two-j shell ($i_{13/2}+g_{9/2}$).

This paper is organized as follows. The particle-rotor model is reviewed in Sec. 2. The methods of spectral statistics are described in Sec. 3. In Sec. 4 the results and discussions are given, and in Sec. 5 the conclusions are summarized.

2 Particle-Rotor Hamiltonian

An even-even nucleus is visualized as an axially symmetric rotor coupled with a few valence particles outside the core. The spin $\vec{I}$ of the system is the sum of the angular momentum $\vec{R}$ of the core and $\vec{J}$, the angular momenta of the valence particles. The total Hamiltonian is divided into two parts,

$$H_{\rm PRM} = H_{\rm intr} + H_{\rm coll}\,, \tag{1}$$

$$H_{\rm intr} = \sum_{j'm'jm} \langle j'm'|H_{\rm sp}|j\ m\rangle a^\dagger_{j'm'} a_{j\ m} + \frac{1}{4}\sum_{j'_1m'_1j'_2m'_2j_1m_1j_2m_2} V^{(2)}_{j'_1m'_1j'_2m'_2j_1m_1j_2m_2} a^\dagger_{j'_1m'_1} a^\dagger_{j'_2m'_2} a_{j_2m_2} a_{j_1m_1}\,, \tag{2}$$

*The project supported by National Natural Science Foundation of China under Grant Nos 19835010 and 10047001, the Major State Basic Research Development Program under contract No. G2000-0774-07, and the Knowledge Innovation Project of the Chinese Academy of Sciences under contract No.KJCX2-SW-No. 2

where $H_{\rm intr}$ describes microscopically the motion of valence particles, $H_{\rm sp}$ is the single particle Hamiltonian for the potential of axially symmetric harmonic oscillator. We take two-body interaction $V^{(2)}$ as delta force $-G\delta(\vec{r_1}-\vec{r_2})$, which can be characterized by the interaction strength G. Here we make the restriction that for the single-j case, the particles are put in single-j orbitals (such as $i_{13/2}$) and for the two-j case the particles can occupy two-j shell orbitals (such as $i_{13/2}$ and $g_{9/2}$, $g_{7/2}$ and $d_{5/2}$). For the rest of the paper, unless stated, single-j shell refers to $i_{13/2}$. In the single-j case,[6]

$$\sum_{j'm'j\,m}\langle j'm'|H_{sp}|jm\rangle a^{\dagger}_{j'm'}a_{jm}=\sum_{m}\kappa\frac{3m^2-j(j+1)}{j(j+1)}a^{+}_{jm}a_{jm}\,, \tag{3}$$

where κ is the energy parameter related to single-particle quadrupole deformation potential, which is different in different shells. In the two-j shell, there are off-diagonal matrix elements between the two-j shells in Eq. (3). In Ref. [7], these off-diagonal matrix elements were not considered. It will be seen from this paper that they play an important role in affecting the degree of chaoticity of the system in the two-j case.

The collective motion of the core is written as

$$H_{\rm coll}=\sum_{i=1}^{3}\frac{R_i^2}{2\mathcal{J}_i}=\frac{I^2-I_3^2}{2\mathcal{J}}+\frac{j_1^2+j_2^2}{2\mathcal{J}}-\frac{I_1j_1+I_2j_2}{\mathcal{J}}=H_{\rm rot}+H_{\rm rec}+H_{\rm cor}\,, \tag{4}$$

where $H_{\rm rot}=(I^2-I_3^2)/2\mathcal{J}$ indicates the nuclear collective rotation, $H_{\rm rec}=(j_1^2+j_2^2)/2\mathcal{J}$ is the recoil term and $H_{\rm cor}=-(I_1j_1+I_2j_2)/\mathcal{J}$ is the coriolis term.

The eigenfunctions of particle-rotor Hamiltonian are

$$\varphi^i_{IM}=\sum_k C^i_k\psi_{IMK}\,, \tag{5}$$

where ψ^{α}_{IMK} is the symmetrized state,

$$\psi^{\alpha}_{IMK}=\sqrt{\frac{2I+1}{16\pi^2(1+\delta_{K0})}}\{\mathcal{D}^{I*}_{MK}(\Omega)\phi^{(1,2,\ldots,N)}_{K\alpha}+(-)^{I+K}\mathcal{D}^{I*}_{M-K}(\Omega)\phi^{(1,2,\ldots,N)}_{\bar{K}\alpha}\}\,, \tag{6}$$

$D^I_{MK}(\Omega)$ is the usual rotation matrix and Ω describes the orientation of the core with respect to space-fixed axes. $\phi^{(1,2,\ldots,N)}_{K\alpha}$ is the N-body antisymmetric wavefunction for the valence particles and α is for the other quantum numbers.

3 Evaluation of Spectral Statistics

Levels of given spins are obtained by diagonalizing the Hamiltonian in Eq. (1). To remove the influence of the local fluctuation of level density on level spacings, we map the spectra $\{E_i\}$ onto the spectra $\{X_i\}$ through $X_i=\bar{N}(E_i)$, where X_i are the unfolded levels. The unfolding of the spectra $\{E_i\}$ are carried out according to the following procedures:[8] choosing an interval of levels with E_i, E_{i+1}, $\ldots$, E_{i+n} (e.g. $n=7$), we can calculate their average spacing: $d_i=\sum_{k=1}^{n}(E_{i+k}-E_{i+k-1})/n=(E_{i+n}-E_i)/n$, then the unfolded levels X_i can be defined as $X_i=E_i/d_i$. The same procedure is repeated until all the levels are covered. The level spacing of the unfolded levels is defined as $S_i=X_{i+1}-X_i$.

The first spectral statistics we studied is the nearest-neighbor distribution $P(S)$. It is obtained by counting the spacings S_i that lie in a certain interval $(S,S+{\rm d}S)$ and normalizing the resulting distribution. We considered the nearest-neighbor distribution (NND) in the interval $S\in(0,2)$.

Spectral statistics shows Poissonian and GOE forms for fully ordered and chaotic systems. One interpolation formula between these two distributions was proposed by Berry–Robnik,[9]

$$P_{\rm BR}(q,S)=\bar{q}^{\,2}\exp(-\bar{q}S)\ {\rm erfc}\left(\frac{1}{2}\sqrt{\pi}qS\right)+\left(2q\bar{q}+\frac{1}{2}\pi q^3S\right)\exp\left(-\bar{q}S-\frac{1}{4}\pi q^2S^2\right)\,, \tag{7}$$

where ${\rm erfc}(x)=1-{\rm erf}(x)$ and ${\rm erf}(x)$ is the error function.

The other interpolation formula is Brody distribution:[10]

$$P_{\rm B}(b,S)=(1+b)AS^b\exp(-AS^{1+b})\,, \tag{8}$$

where

$$A=\left\{\Gamma\left(\frac{2+b}{1+b}\right)\right\}^{1+b} \tag{9}$$

and Γ is the usual gamma function. $q=b=0$ corresponds to the Poisson distribution, while $q=b=1$ corresponds to the GOE distribution.

NND emphasizes short-range correlation characteristics. The long-range correlation is measured by the spectral rigidity $\Delta_3(L)$, which is defined as

$$\triangle_3(\alpha,L)=\frac{1}{L}\int_{\alpha}^{\alpha+L}[N(E)-(AE+B)]^2\,{\rm d}E\,, \tag{10}$$

where A and B are fitting parameters, α and $\alpha+L$ describe the order number of the unfolded levels. To get $\bar{\triangle}_3(L)$, we proceed as described in Ref. [11]. For a given stretch of levels, we calculate $\triangle_3(\alpha,L)$ for the interval $[\alpha,\alpha+L]$, $[\alpha+L/2,\alpha+3L/2]$, $[\alpha+L,\alpha+2L]$, $[\alpha+3L/2,\alpha+5L/2]$, $\ldots$

until the stretch $[a,b]$ has been covered, then we average α to get

$$\bar{\triangle}_3(L)=\frac{1}{N'}\sum_i \triangle_3\Big(\alpha+\frac{i-1}{2}L,L\Big)\,, \qquad (11)$$

where N' is the number of sums in the numerator.

4 Results

In our calculations six particles were put in a single $j=13/2$ orbital in the single-j case, while in the two-j case six particles occupied two-j ($i_{13/2}+g_{9/2}$ or $g_{7/2}+d_{5/2}$) orbitals. The spectral statistics were carried out using states with a given spin in the particle-rotor model. Unless stated we used the following parameters in the calculation: $G=G_0=0.45$ MeV, $\mathcal{J}=\mathcal{J}_0=24\hbar^2$ MeV,$^{-1}$ and $\kappa=\kappa_0=2.5$ MeV, 2.4 MeV, 2.2 MeV, and 2.0 MeV for $i_{13/2}$, $g_{9/2}$, $g_{7/2}$ and $d_{5/2}$ orbitals.

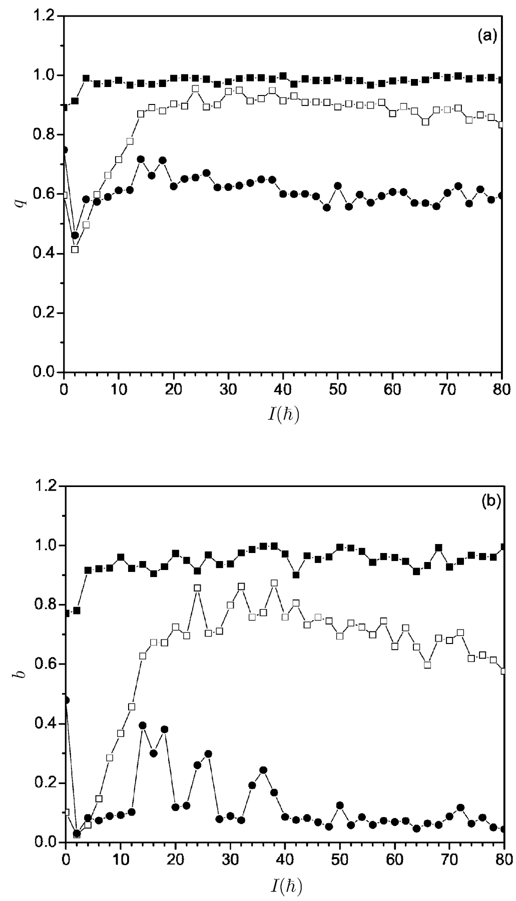

Fig. 1 The best-fit parameters for the degree of chaoticity with mixed statistics are shown for (a) Berry–Robnik parametrization, (b) Brody parametrization. The results for single-j shell $i_{13/2}$, two-j shell $(g_{7/2}+d_{5/2})$, and two-j shell $(g_{7/2}+d_{5/2})$ without considering the off-diagonal matrix elements in Eq. (3) are indicated by solid square, open square, and solid circle, respectively.

In Ref. [5] chaotic behavior of PRM in single $j=13/2$ space was studied. Now we do not change the dimension of configuration space and consider two-j shell $(g_{7/2}+d_{5/2})$ instead of single-j shell. The total basis size is 1519 for high even angular momenta (signature +1) in PRM. This number is smaller in the single-j case for $I<24$, e.g., for $I=20$ one has 1512 states and it falls to just 93 configurations for $I=0$. In the two-j case $(g_{7/2}+d_{5/2})$, this number is smaller for $I<12$, e.g., for $I=10$ the number is 1513 and 165 for $I=0$.

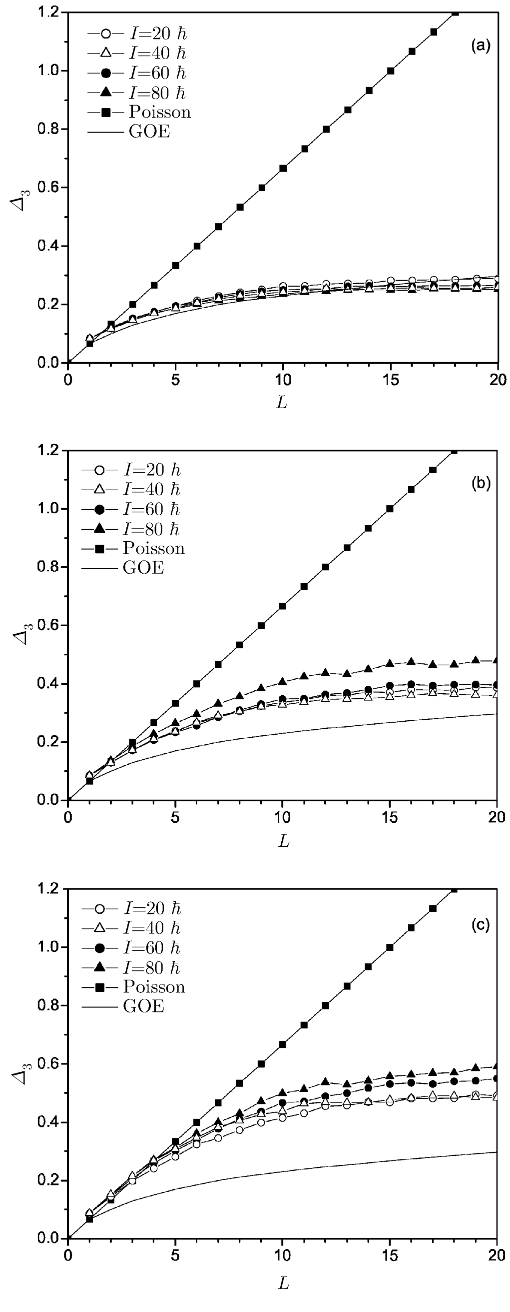

Fig. 2 The spectral rigidity of the particle-rotor Hamiltonian is presented for different values of spins in (a) single-j shell ($i_{13/2}$), (b) two-j shell $(g_{7/2}+d_{5/2})$, and (c) two-j shell $(g_{7/2}+d_{5/2})$ without considering the off-diagonal matrix elements in Eq. (3).

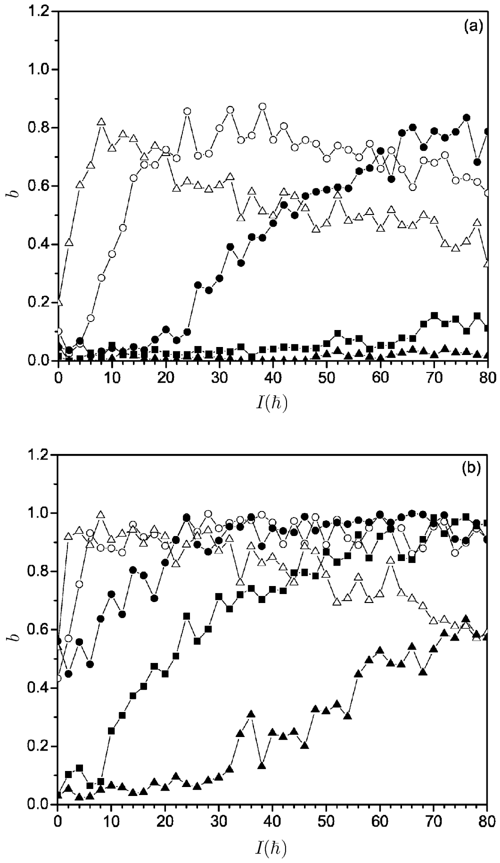

Fig. 3 The degree of chaoticity in (a) two-j shell ($g_{7/2}+d_{5/2}$) and (b) two-j shell ($i_{13/2}+g_{9/2}$) is displayed as a function of spin for different values of the moment of inertia $\mathcal{J}$. The results for $\mathcal{J}=8, 24, 72, 216$, and $648\hbar^2$ MeV^{-1} are indicated by open triangle, open circle, solid circle, solid square, and solid triangle, respectively.

The calculated NND in single-j and two-j shell ($g_{7/2}+d_{5/2}$) were fitted with both the formulas (7) and (8). In Fig. 1 we show the best-fit Berry–Robnik and Brody parameters as functions of the spin. It was pointed out[5] that although the quantitative predictions for the degree of chaoticity were quite different for these two kinds of parameters, the qualitative behavior was the same, which can be seen from Figs 1(a) and 1(b). For the rest of this paper we use the NND statistics of the Brody parametrization. In Fig. 1 we notice that the Berry–Robnik parameter q and Brody parameter b are larger in the single-j case than those in the two-j case, which means that the system becomes more regular when the configuration space is changed from single-j to two-j shell ($g_{7/2}+d_{5/2}$). This conclusion accords with that in Ref. [7], where spectral statistics were studied in the framework of interacting boson model (IBM).[13] In the present study it can be explained as follows: in the single-j case, the chaos of the system is caused by the coriolis force, delta interaction and two-body recoil term. However, all these off-diagonal elements which result in chaos only exist between two states whose quantum numbers j are the same. So, when the space is changed from single-j to two-j shell ($g_{7/2}+d_{5/2}$), the total number of states is unchanged and although new off-diagonal matrix elements in Eq. (3) appear, the total number of off-diagonal elements decreases for the single-j case, which decreases the degree of chaoticity of the system.

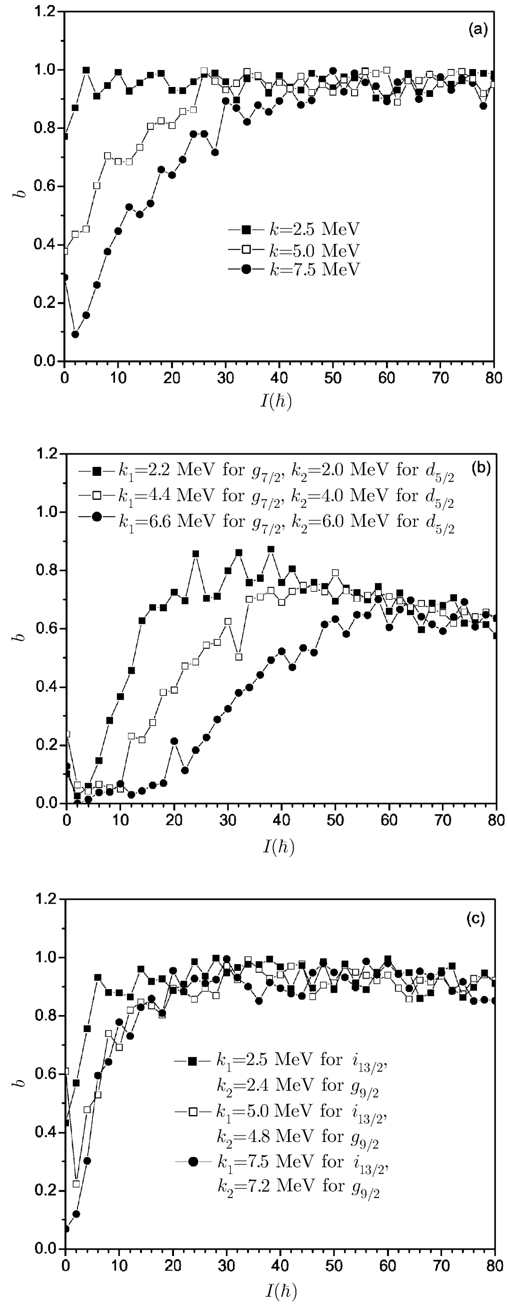

Fig. 4 The degree of chaoticity is depicted as a function of the spin in (a) single-j shell ($i_{13/2}$), (b) two-j shell ($g_{7/2}+d_{5/2}$), and (c) two-j shell ($i_{13/2}+g_{9/2}$).

We also studied the effects of these off-diagonal matrix elements in Eq. (3) on the degree of chaoticity of the system in the two-j case. The Berry–Robnik and Brody parameter values as functions of the spin in the two-j case $(g_{7/2}+d_{5/2})$ without considering the off-diagonal elements in Eq. (3) are also given in Fig. 1. In this case, the system becomes more regular because these off-diagonal matrix elements disappear which couple the two subspaces in the two-j case and make the system more chaotic. This phenomenon can also be seen from Fig. 5, where the single-j shell is extended to two-j shell $(i_{13/2}+g_{9/2})$.

The spectral rigidity in PRM in the cases of single-j and two-j shells is displayed for different spins in Fig. 2. The same conclusion can be drawn. That is, the system becomes more regular when the space is changed from single-j shell to two-j shell $(g_{7/2}+d_{5/2})$ and the off-diagonal elements of single-particle Hamiltonian which couple these two single-j subspaces in the two-j case play an important role in increasing the degree of chaoticity of the system.

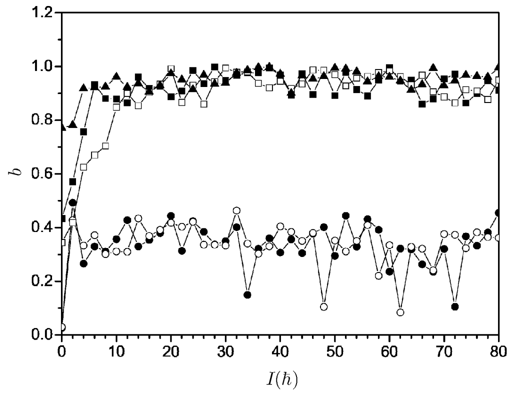

Fig. 5 The same as Fig. 1(b), but for the two-j shell $(i_{13/2}+g_{9/2})$. The results in the single-j shell, two-j shell $(i_{13/2}+g_{9/2})$ with truncation energy $E=18.5$ and 19.0 MeV, and in the two-j shell $(i_{13/2}+g_{9/2})$ without considering the off-diagonal matrix elements in Eq. (3) with truncation energy $E=18.5$ and 19.0 MeV are indicated by solid triangle, open square, solid square, open circle, and solid circle. The results corresponding to different truncation energy are nearly the same, which indicates that our results are reliable.

The effect of the moment of inertia is investigated by changing $\mathcal{J}$ and fixing the values of the other parameters. In Ref. [5] this effect was studied in the single-j case, and their conclusion, that is, the degree of chaoticity will decrease when the spin increases, holds true only for smaller values of $\mathcal{J}$, and the opposite trend prevails for large moments of inertia. We investigate the effect of the moment of inertia in the two-j case $(g_{7/2}+d_{5/2})$. The results are shown in Fig. 3(a). The same conclusion can be drawn as Ref. [5]. Figure 3(b) can also give the same conclusion, where the single-j shell is extended to the two-j shell $(i_{13/2}+g_{9/2})$.

We next vary the deformation parameter and fix other parameters in the standard parameters. The results in single-j shell, two-j shell $(g_{7/2}+d_{5/2})$, and two-j shell $(i_{13/2}+g_{9/2})$ are shown in Fig. 4. For large κ, a decrease in chaoticity can be observed. In the single-j case the degree of chaoticity becomes independent of κ quickly, while this convergence is slower in two-j shell $(g_{7/2}+d_{5/2})$ and quicker in two-j case $(i_{13/2}+g_{9/2})$.

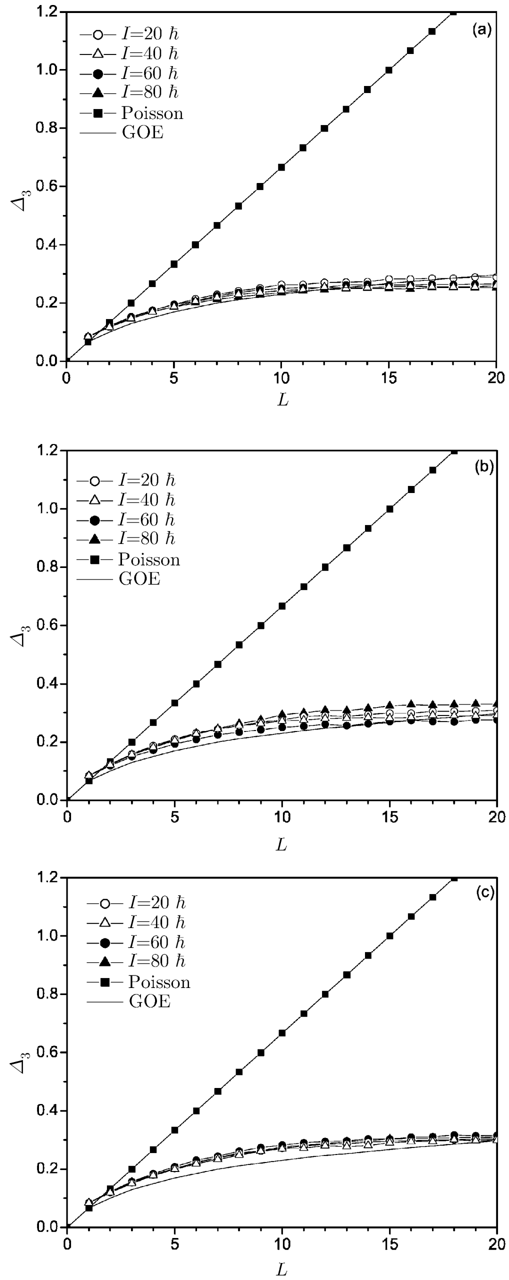

Fig. 6 The same as Fig.2, but for two-j shell $(i_{13/2}+g_{9/2})$.

Now we want to see what will happen if we enlarge the configuration space by extending single-j shell $(i_{13/2})$ to two-j shell $(i_{13/2}+g_{9/2})$. In the two-j case $(i_{13/2}+g_{9/2})$, configuration truncation is used. In our calculation, we took the truncation energy for valence particles as $E=18.5$ MeV and $E=19.0$ MeV and the corresponding basis size is 1667 and 2110 when $I\geq 24$.

The fitted Brody parameters as the functions of spin for single-j shell and two-j shell ($i_{13/2}+g_{9/2}$) are given in Fig. 5. It is shown that a few changes happen when we enlarge the configuration space. The results of two kinds of truncation are nearly the same, which indicates that our truncation is reliable. The same conclusion can be drawn from Fig. 6 where the spectral rigidity in single-j and two-j ($i_{13/2}+g_{9/2}$) cases is shown.

5 Conclusions

A systematic study of the chaotic behavior in two realistic models has been carried out. The difference of NND and spectral rigidity between single-j case ($i_{13/2}$) and two-j case ($g_{7/2}+d_{5/2}$) shows that the system is more regular in the two-j model ($g_{7/2}+d_{5/2}$) than that in the single-j case ($i_{13/2}$), although the basis size of the configuration space is unchanged. However, when the single-j shell space is enlarged by extending single-j shell to two-j shell ($i_{13/2}+g_{9/2}$), the degree of chaoticity of the system changes slightly. In the two-j case, the conclusion[5] also holds true that chaos is more pronounced for normal deformation and low spins than for superdeformations and high spins.

It is known that the two-body interaction plays an important role in the appearance of chaotic motion.[14,15] The present study, however, reveals that the off-diagonal matrix elements of one-body interaction (single-particle Hamiltonian) also play an important role in affecting the degree of chaoticity of the system.

Acknowledgments

We are grateful to H.D. Heiss, I. Hamamoto, Jian-Zhong GU, Jun-Qing LI, and Ren-Rong ZHENG for their helpful discussions and suggestions.

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